NOVEL CUTS OF TRIPLY-ROTATED QUARTZ CRYSTAL FOR RESONATORS WITH IDEAL CUBIC FREQUENCY-TEMPERATURE RELATIONS

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A systematic method of searching novel cuts of quartz crystal is proposed to explain the discovery of the existing cuts including AT- and SC-cut, and a few popular cuts in products on market. It is also found that some curves on which the cuts have the same frequency-temperature relation as that of the AT- and SC-cut quartz resonators and the inflection temperature cover a quite wide range. Finally, using this method, we found a novel cut whose frequency-temperature relation is much better than the AT- and SC-cut quartz crystal resonators. The temperature range that the absolute value of frequency shifts is smaller than 10ppm for this novel cut from $-70℃$ to $130℃$ and the frequency constant is 35% higher than the AT-cut resonator.

Keywords: Frequency; Temperature; Cut; Quartz; Resonator

1. INTRODUCTION

Quartz belongs to anisotropic crystals, so quartz crystal resonators whose core components are plates perform differently when the plates are cut in different orientations. As a result, we can find some special orientations that resonators show useful properties such as temperature insensitivity. From the invention of resonators up to now, many cuts have been found and utilized commercially such as AT- and SC-cuts. The complete list of the cuts can be found in reference books and web sources.

Since names of most cuts are related to temperature, temperature is the most important factor that determines the quality of quartz resonator. For the quartz resonator that is used in timing and frequency control, the vibration frequency of resonator is expected to be insensitive with the temperature variation. For the quartz resonator that is used as a temperature sensor, linear frequency-temperature relation is always better.

Although great achievements have been made by efforts of pioneers, following questions from product development engineers are to be answered:

1) Can we explain the existing cuts systematically?
2) Can we find other cuts with frequency-temperature behavior better than the known AT- and SC-cut?
3) Can we find cuts whose frequency-temperature behavior is more stable than AT- and SC-cut?

2. METHODS

2.1. Frequency-temperature relations

The vibration frequency of a resonator shifts when the environment temperature varies. The relation of a cut whose rotation angles are denoted as $(yxwlw)\phi,\theta,\psi$ according to the standard IRE 49[1] can be described by a function of frequency shift $\Delta f/f$ vs. temperature T [2]

$$\begin{gathered}\frac{\Delta f}{f}(\phi,\theta,\psi,T)=T_f^{(1)}(\phi,\theta,\psi,T_0)(T-T_0)+\\ T_f^{(2)}(\phi,\theta,\psi,T_0)(T-T_0)^2+T_f^{(3)}(\phi,\theta,\psi,T_0)(T-T_0)^3\end{gathered}\tag{1}$$

where $T_f^{(n)}$ is the nth-order frequency-temperature coefficients and T_0 is the reference temperature. Only the first three terms of Taylor series are retained because the frequency-temperature relation is cubic in experiment.

Generally, a function $f(\phi,\theta,\psi,T)$ can be derived for a special vibration mode such as the flexural,

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thickness-shear, face-shear mode, and others [3]. For example, the derivation of $f(\phi,\theta,\psi,T)$ of thickness-shear mode of an infinite plate with the incremental thermal field theory can be found in an early paper [4].

If the function $f(\phi,\theta,\psi,T)$ is known, then the first three frequency-temperature coefficients can be obtained by the finite differential methods [5].

The thermal elastic constants and thermal expansion coefficients are needed to calculate the frequency-temperature relation. Since they are correct only in certain temperature range, we need to determine which range of temperature are reliable for the analysis. The physical constants used here are from Lee et al. [6] who used the data measured by Mason [7], Bechmann [2], Adams [8], and Kahan et al. [9]. The range of temperature measured by Mason, Bechmann, and Adams are $(-100℃, 200℃)$, $(-196℃, 100℃)$, $(-35℃, 100℃)$ respectively, and it's not shown by Kahan. Strictly speaking, the range of temperature should not exceed $(-196℃, 200℃)$ according to the experiment, but the physical constants do not deviate much beyond the phase transition between α- and β-quartz at $573℃$. Furthermore, Patel extended the temperature to $300℃$ when he was estimating the Q factor [10]. Thus, temperature is mapped into $(-196℃, 300℃)$ in this study.

3. RESULTS

The existing cuts can be confirmed by solving the equations of the first third-order frequency-temperature coefficients of flexural, longitudinal, thickness-shear and face-shear mode in singly-, doubly- and triply-rotated quartz. According to different orientation, we discuss the following cases.

Case 1

$$T_f^{(1)}\left(0°,\theta,0°,25℃\right)=0, \tag{2}$$

where θ is the singly-rotated angle $(yxl)\theta$. The roots of the Eq. (2) are denoted as θ_t , and the frequency-temperature relation at θ_t is

$$\frac{\Delta f}{f}=T_f^{(2)}(0°,\theta_t,0°,25℃)(T-25℃)^2+ \\ T_f^{(3)}(0°,\theta_t,0°,25℃)(T-25℃)^3. \tag{3}$$

Because of the Taylor expansion, $T_f^{(3)}$ is always smaller than $T_f^{(2)}$ near the reference temperature $25℃$, and the frequency-temperature often show the parabolic form.

With this idea and solving the Eq. (2) of different vibration modes of singly-rotated quartz, the AT-, BT- [11], RT- [2], AK-cut [12] of quartz crystal resonators of thickness-shear mode, CT- and DT- cuts [13] of face-shear mode, NT-cut [14] of flexural mode, ET- and FT- cuts [13] probably of second flexural mode, GT- and MT- cuts [13,14] of longitudinal mode are found.

Case 2

$$T_f^{(1)}(\phi,\theta,0°,25℃)=0, \tag{4}$$

where ϕ,θ are the doubly-rotated angles $(yxwl)\phi,\theta$. There are two variables and one equation, so the roots ϕ_t,θ_t of the Eq. (2) are on curves and the corresponding cuts have parabolic frequency-temperature relations. These curves of B and C mode of thickness vibrations were first derived in the Fig. 2 of Bechmann's paper in 1962 [2]. The curve of C mode is also the BBLC1 in Figure 4 below.

Case 3

$$\begin{cases} T_f^{(1)}(\phi,\theta,0°,T_0)=0, \\ T_f^{(2)}(\phi,\theta,0°,T_0)=0. \end{cases} \tag{5}$$

Different from Eq. (4), $T_f^{(1)}$ and $T_f^{(2)}$ are functions

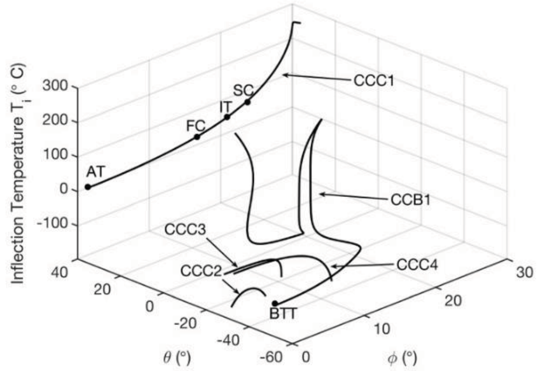

Figure1. Orientations with frequency-temperature relations exhibit ideal cubic function. CCC1-4 means the first, second, third and fourth curve of cubic frequency-temperature function of C mode. CCB1 means the first curve of cubic frequency-temperature function of B mode.

of the reference temperature T_0. It means that the reference temperature T_0 is determined by solving the

Eq. (3). The roots are denoted as ϕ_i, θ_i, T_i, which means that the quartz resonators of doubly-rotated cuts ϕ_i, θ_i have the frequency-temperature relation

$$\frac{\Delta f}{f} = T_f^{(3)}(\phi_i, \theta_i, 0°, T_i)(T - T_i)^3. \tag{6}$$

This frequency-temperature relation is the same as that of AT-, FC-, IT- and SC-cut [15] whose rotation angles and inflection temperature are listed in Table 1, and we found the angles and inflection of these cuts are all on the curves of ϕ_i, θ_i, T_i in Fig. 1.

Table 1. Orientations and inflection temperature of AT-, FC-, IT-, SC and BTT-cut

Cuts	ϕ (°)	θ(°)	T_i(°C)
AT-cut	0	35.25	25
FC-cut	15	34.33	50
IT-cut	19.1	34.08	75
SC-cut	21.93	33.93	95
BTT-cut [16]	0	-51.993	-100

Case 4

We can also consider the stress function $\sigma(\phi, \theta, \psi)$ [17] between the electrode and the crystal plate. By solving the equations

$$\begin{cases} \sigma(\phi, \theta, 0°) = 0, \\ T_f^{(1)}(\phi, \theta, 0°, 25°\text{C}) = 0 \end{cases} \tag{7}$$

of C mode, EerNisse [18,19] found the SC-cut resonator first. In fact, according to the Eq. (3), the frequency-temperature relation is parabolic while it is ideally cubic in commercial applications. Strictly speaking, the SC-cut should be found from curves of stress equation and of Eq. (5). However, since the curve of roots of the Eq. (2) of C mode is very close to the curve of roots of Eq. (5) of C mode in Fig. 2, Kuster [20] found SC-cut with ideal cubic frequency-temperature relation near the initial result of EerNisse experimentally.

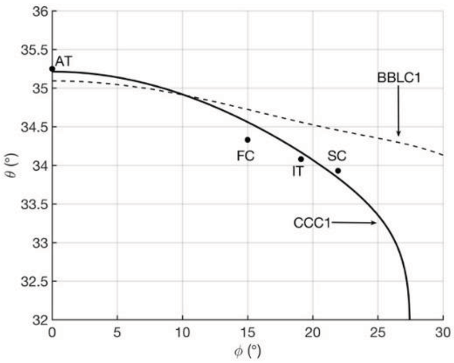

Figure 2. Optimal cuts of quartz crystal with cubic frequency-temperature relation. The solid line is the solution of Eq. (5) and the dash line is the solution of Eq. (4).

In addition, solving the stress equation of B mode and first frequency temperature coefficients equation of C mode, Valdios obtained the SBTC-cut [21] that have stress compensation of B mode and temperature compensation of C mode.

Case 5

$$\begin{cases} T_f^{(2)}(\phi, \theta, 0°, T_0) = 0, \\ T_f^{(3)}(\phi, \theta, 0°, T_0) = 0. \end{cases} \tag{8}$$

If the second and third frequency-temperature coefficients are zeroes, the frequency-temperature relation of the corresponding roots ϕ_l, θ_l, T_l show the linear relation

$$\frac{\Delta f}{f} = T_f^{(1)}(\phi_l, \theta_l, 0°, T_l)(T - T_l). \tag{9}$$

Such cuts can be used for thermometers, and Hammond invented the LC-cut [22] quartz resonator based on this method.

Case 6

$$\begin{cases} \sigma(\phi, \theta, 0°) = 0, \\ T_f^{(2)}(\phi, \theta, 0°, 25°\text{C}) = 0. \end{cases} \tag{10}$$

Nakazawa considered the stress equation and the equation of the second-order frequency-temperature coefficient. He found the NLSC-cut [23] whose angles are denoted as ϕ_{NL}, θ_{NL} for thermometer that is stress insensitive and has the frequency temperature relation

$$\frac{\Delta f}{f} = T_f^{(1)}(\phi_{NL}, \theta_{NL}, 0°, 25°\text{C})(T - 25°\text{C}) + T_f^{(3)}(\phi_{NL}, \theta_{NL}, 0°, 25°\text{C})(T - 25°\text{C})^3. \tag{11}$$

Although Eq. (3) contains the cubic term, the frequency-temperature relation is very close to the linear relation near 25°C because the third-order temperature coefficient is much smaller than the first-order temperature coefficient.

Case 7

$$\begin{cases} T_f^{(1)}(\phi,\theta,\psi,T_0) = 0, \\ T_f^{(2)}(\phi,\theta,\psi,T_0) = 0, \\ T_f^{(3)}(\phi,\theta,\psi,T_0) = 0. \end{cases} \tag{12}$$

We can consider the extreme case that all the first third-order temperature coefficients are zeroes so that the frequency-temperature relation is

$$\frac{\Delta f}{f} = 0, \tag{13}$$

which means the frequency of a resonator does not vary when the environment temperature changes. Obviously, it is the cut with best frequency-temperature relation.

The solution of Eq. (12) can be transformed to search minimum of objective function

$$F = {T_f^{(1)}}^2 + {T_f^{(2)}}^2 + {T_f^{(3)}}^2. \tag{14}$$

Then check if the Eq. (12) is satisfied.

Unfortunately, we cannot find any cut with Eq. (13). However, if F is smaller than a small tolerance we can think of, the minimum is the root of Eq. (6) strictly. If F is smaller than a small tolerance, we can find the frequency-temperature relation varies very slow too.

With this approach, we found a cut whose frequency-temperature relation is much flatter than that of AT- and S-cut as shown in Fig. 3. This cut is a triply-rotated cut, and its vibration mode is B mode of thickness-shear. The frequency constant is 35% higher than AT-cut. The inflection temperature is $25℃$, and the range that the absolute value of frequency shift is smaller than 10ppm is from $-70℃$ to $130℃$.

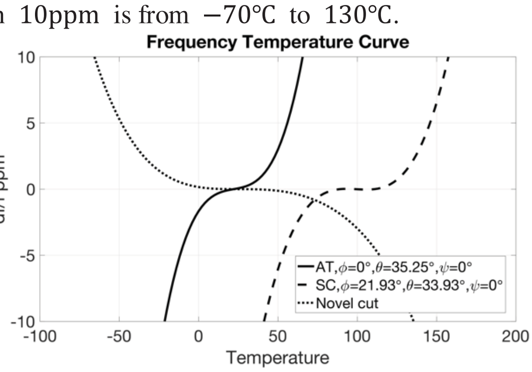

Figure 3. Frequency-temperature relation of novel cut, AT-cut and SC-cut.

4. CONCLUSIONS

A systematic method is presented to explain the discovery of the existing cuts of quartz crystal. We have found more cuts with the same or superior frequency-temperature relation of AT- and SC-cut. Finally, we suggested a novel cut that have a much stable frequency-temperature behavior with great potential in devices that work in a wide range of temperature.

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